# Three-Port 3-dB Power Divider Terminated by Arbitrary Impedances

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## Abstract

The three-port power divider terminated by arbitrary impedances considered in this paper is very useful for small-sized circuit design. New design equations for three-port power divider are derived. They can be applied to the three-port power dividers with both arbitrary termination impedances and especially a 3-dB power division. On the basis of these design equations, simulations for the three port 3-dB power divider terminated by 45, 30 and 50 Ω were made using ideal transmission lines. Also, a microstrip three-port 3-dB power divider terminated by 30, 53 and 47 Ω was fabricated on $Al_2O_3$ substrate ($\varepsilon_r$ = 10 and h = 635 μm) and it shows good agreement between experimental results and theoretical results.

## Introduction

A large number of different types of power dividers, with and without isolation between the output ports are used for various applications. Certain power dividers, which provide isolation between output ports, either have quadrature outputs, or in-phase or out-of-phase outputs with respect to the input port.

As a special class of couplers, so-called hybrids, among the power dividers which provide in-phase or out-of-phase responses are ring hybrids [1]-[2] and three-port-power dividers [3]-[5],. [7]-[9]. Power dividers which provide quadrature outputs consist of branch-line hybrids [6].

In case that three-port power dividers or four-port power dividers can be used with active elements and/or other passive elements, additional matching networks are necessary to obtain the desired output performances. In these cases, if these power dividers are terminated by arbitrary impedances, the total size of integrated microwave circuits can be reduced.

Four-port power dividers terminated by arbitrary impedances, for the first time, have been treated by Ahn et *al* who investigated ring hybrids [1]-[2]. For three-port power dividers terminated by arbitrary impedances, there are two literature references [8], [9]. However, in [8], special load impedances, where even and/or odd analyses are possible, are used and in case of 3-dB power division, all termination impedances must be the same. [9] treated the three-port power divider terminated by complex frequency-dependent impedances. However, this three-port power divider is effective only using optimization techniques and it can be analyzed just in the case of available mirror reflection symmetry. The paper does not present design equations as well.

In this paper, design equation for a three-port power divider with both arbitrary termination impedances and 3-dB power division will be described. On the basis of the design equations, a three-port power divider terminated by 45, 30 and 50 Ω was analyzed under ideal conditions. Additionally, a microstrip three-port 3-dB power divider terminated by 30, 53 and 47 Ω was fabricated on $Al_2O_3$ substrate ($\varepsilon_r$ = 10 and h = 635 μm).

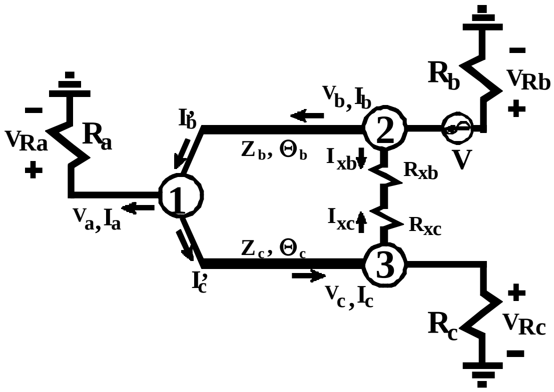


Fig. 1. Power divider terminated by arbitrary impedances.

# Analyses

Wilkinson described a device that separated one signal into n equiphase-equiamplitude signals ($n \geq 2$). With n = 2, his circuit can be reduced to a three-port 3-dB power divider terminated by equal impedances. However, Fig. 1 shows a three-port 3-dB power divider terminated by arbitrary impedances $R_a$, $R_b$ and $R_c$ .

From transmission line equations,

node ② and ①: $V_b = V_a \cos\Theta_b + jI_b' Z_b \sin\Theta_b$ (1)

$$I_b = I_b' \cos\Theta_b + j\frac{V_a}{Z_b}\sin\Theta_b \quad (2)$$

node ③ and ①: $V_a = V_c \cos\Theta_c + jI_c Z_c \sin\Theta_c$ (3)

$$I_c' = I_c \cos\Theta_c + j\frac{V_c}{Z_c}\sin\Theta_c \quad (4)$$

From node equations,

node ①: $I_b' = \frac{V_a}{R_a} + I_c'$ (5)

node ②: $I_c = \frac{V_c}{R_c} + I_{xc}$ (6)

node ② and ground: $(I_b + I_{xb})R_b = V - V_b$ (7)

node ② and node ③: $I_{xb}R_{xb} - I_{xc}R_{xc} = V_b - V_c$ (8)

If this divider is a 3-dB power divider, the power excited at port ① must be transferred equally to load ② and load ③. That means $(V_{Rb}\ V^*_{Rb})/R_b = (V_{Rc}\ V^*_{Rc})/R_c$.

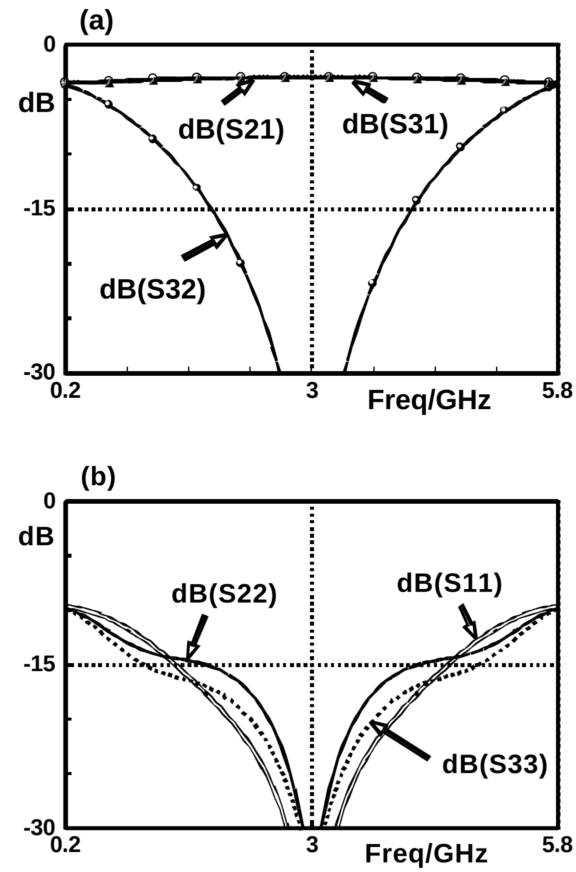


Fig. 2. Simulated results for a three-port 3-dB power divider terminated by 45, 30 and 50Ω .
(a) Power division and isolation characteristic,
(b) all-port reflection coefficients.

In case of $R_b \neq R_c$ , $V_{Rb}$ is not equal to $V_{Rc}$. Also $V_{Rb} \neq V_{Rc}$ means that undesired power dissipates between node ② and node ③. To reduce the dissipated power, the supposed load can be made as an optimal load, namely $R_{av}$ .

Applying $R_{av}$ and $\Theta_a = \Theta_b = \frac{\pi}{2}$ to (1) - (8) and through calculations for perfect isolation, $V_c = 0$ , we can get

$$\frac{V_b}{V} = \frac{\frac{1}{R_{av}}}{\frac{2}{R_{xc}+R_{xb}} + \frac{1}{R_{av}}} = \frac{1}{1 + \frac{2R_{av}}{R_{xc}+R_{xb}}}. \quad (9)$$

From (9), transmission line characteristic impedances $Z_b$, $Z_c$ and isolation resistor R can be derived as follows:

$$Z_b = \sqrt{(R_b + R_c)R_a}\,, \qquad (10)$$
$$Z_c = \sqrt{(R_b + R_c)R_a}\,,$$
$$R = R_{xc} + R_{xb} = 2R_{av}\,.$$

On the basis of the derived design equations, a three-port 3-dB power divider terminated by 45, 30 and 50 Ω, was analyzed assuming ideal transmission lines.

Fig. 2(a) shows that the power divison is in worst case -3.524 dB in full bandwidth (0.2 - 5.8 GHz). Isolation is theoretically perfect ($|S_{32}|$ = -152.905 dB at the center frequency of 3 GHz) and better than 15 dB at the edge of 2.1:1 bandwidth as well.

Table ❶ : Microstrip three-port 3-dB power divider.
( center frequency : 3 GHz )

| Termination impedances | Microstrip feeding transformer lines (μm) | Microstrip transmission lines (μm) |
|---|---|---|
| Port ①; $R_a$ = 30 Ω | $Z_{o1}$; 38.73 Ω<br>w = 998.743<br>l = 9377.22 | $Z_b$; 54.77 Ω<br>w = 504.743<br>l = 9688.85 |
| Port ②; $R_b$ = 53 Ω | $Z_{o2}$ ; 51.48 Ω<br>w = 578.049<br>l = 9633.22 | $Z_c$; 54.77 Ω<br>w = 504.743<br>l = 9688.85 |
| Port ③; $R_c$ = 47 Ω | $Z_{o3}$ ; 48.48 Ω<br>w = 653.830<br>l = 9579.36 | |

# Microstrip Three-Port 3-dB Power Divider

On the basis of the presented design equations (1), a microstrip three-port 3-dB power divider was fabricated on $Al_2O_3$ substrate ($\varepsilon_r$ = 10 and h = 635 μm). Its termination impedances and the experimental data of the transmission-lines and transformer-lines are shown in Table ❶. This power divider is not terminated by 50 Ω. Therefore, additionally λ/4 transformers are necessary to measure this power divider properties. $Z_{o1}$, $Z_{o2}$ and $Z_{o3}$ seen in Table ❶ are the characteristic impedances of the microstrip-transformer lines. $Z_b$ and $Z_c$ shown in Table ❶ are characteristic impedances of the transmission lines in Fig. 1.

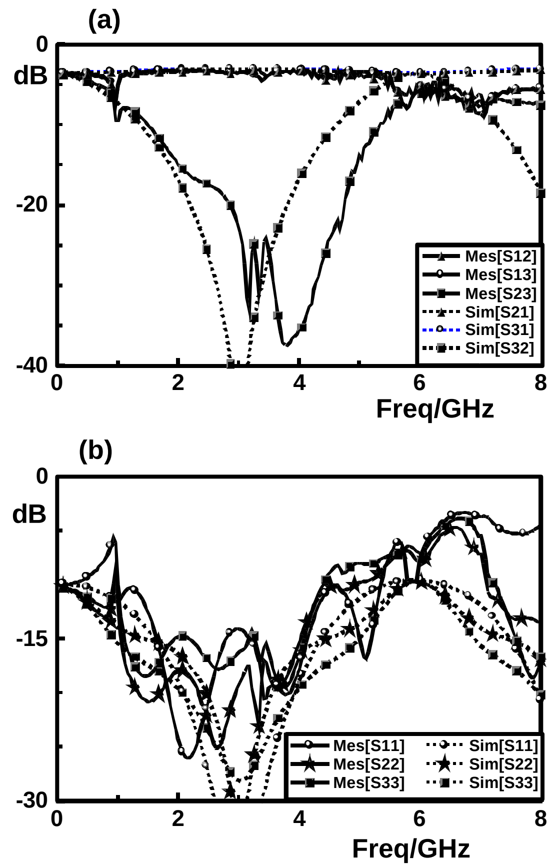


Fig. 3. Measured and simulated results of microstrip three-port 3-dB power divider terminated by 30, 53 and 47 Ω.
(a) Power division and isolation characteristic,
(b) all-port reflection coefficients.

Fig. 3 shows the comparison between simulated and measured results. This power divider was designed at a center frequency of 3 GHz. The experimental results of Fig. 3(a) show, at the bandwidth of 2.5:1, the isolation between two outputs are better than 15 dB and 3 dB power divisions are possible in a bandwith of 0.1 - 5.3 GHz. In Fig. 3(b), all-port reflection coefficients are less than -15 dB in a bandwidth of 2.5:1.

# Conclusion

In this paper, new design equations for three-port 3-dB power divider terminated by arbitrary impedances are presented. Using new design equations, there are large advantages gained by reducing the total size of the integrated microwave circuit.

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